Nearly optimal pulse control of quantum systems

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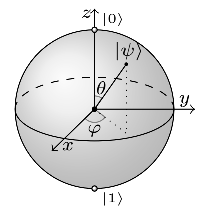
1. Introduction

Quantum control refers to the ability to steer the state or dynamic evolution of a quantum system by means of electromagnetic radiation such as a laser, a magnetic field, *etc.* A quantum system is described by the Schrödinger equation

$$i\hbar \frac{d}{dt}|\Psi(t)\rangle = \hat{H}(t)|\Psi(t)\rangle$$

that is well known to be very sensitive to noise [2]. To deal with this problem and so as to manipulate such system there are several numerical methods and strategies, including pulse control. In this work, we reproduce efficiently and precisely optimal pulses for quantum systems such as those described in [1]. To this end, we rely in particular on the Julia package `OptimalControl.jl` from control-toolbox.org to achieve performance together with a high level and flexible description of the quantum control systems.



2. Methods

Consider the following control system:

$$\dot{q}(t) = f(q(t), u(t), t), \quad \mathcal{C} = \int_{t_0}^{t_f} L(q(t), u(t), t)\, dt + \Phi(q(t_f)),$$

where $x(t) \in \mathbb{R}^n$ is the state (1), $u(t) \in U \subset \mathbb{R}^m$ is the control, and the goal is to minimize the cost $\mathcal{C}$ (2) subject to the initial condition $q(t_0) = q_0$.

- To solve quantum control problems, we leverage classical optimal control methods. Specifically, the Pontryagin Maximum Principle (PMP), which provides the Hamiltonian (3).
- To find the optimal control, we need to maximize the Hamiltonian (4).

Now, we have all information to apply a solver from `OptimalControl.jl`:

- Using a direct solver (discretization of the control problem into a math program) to obtain an initial approximation of the solution;
- Then applying shooting to find the zero of (5) to refine the solution using the initial guess from the direct solver.

3. Definition of the problem

The goal in this case is to minimize energy of the control needed to perform a state-to-state transfer in a two-level quantum system (with fixed final time):

DYNAMICS

$$\begin{cases} \dot{x} &= -\Delta y, \\ \dot{y} &= \Delta x - uz, \\ \dot{z} &= uy \end{cases} \qquad q(0) = (0,0,1)^T, \quad u(t) \in [-1,1] \tag{1}$$

COST

$$\mathcal{C} = \|\langle\uparrow|\psi(t_f)\rangle\|^2 + \frac{\gamma}{2}\int_0^{t_f} u^2(t)\, dt \tag{2}$$

PMP

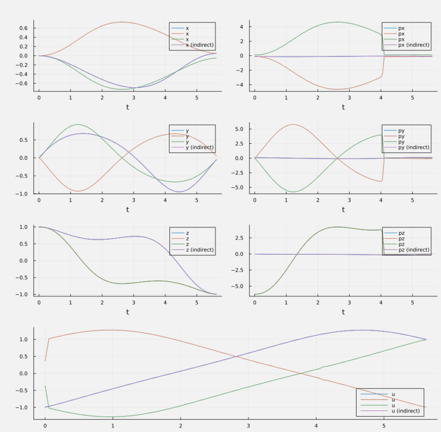
$$H_P = \Delta(p_y x - p_x y) + u(p_z y - p_y z) + \mathcal{V}\frac{\gamma}{2}u^2 \tag{3}$$

$$u = \frac{p_z y - p_y z}{\gamma} \tag{4}$$

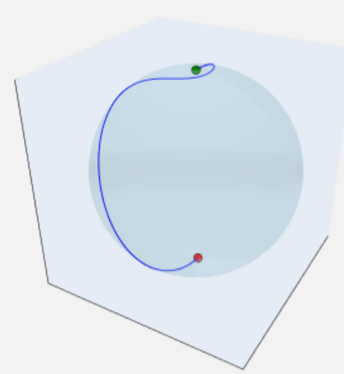
SHOOTING

$$S : \mathbb{R}^3 \longrightarrow \mathbb{R}^3, \quad S(p_0) := p(t_f, q_0, p_0) + 2(q(t_f, x_0, p_f) - q_f) \tag{5}$$

4. Implementation and Numerical Results



The graphs show the evolution of the system against time. The 3D plot illustrates the trajectory of the system in the Bloch sphere. The purple line and blue line (overlapped by the purple line) solutions successfully reach the the qubit $|1\rangle$, confirming the effectiveness of the methods. The other lines are also solutions of the problem and can be deduced from the previous one thanks to certain symmetries. (In practice, they were obtained adding some state constraints to help convergence, for instance bounds such as $|x(t)| \leq 1$, $|y(t)| \leq 1$, $|z(t)| \leq 1$).



5. Conclusion and Future Work

The results obtained with the Julia package OptimalControl.jl match the expected theoretical values. In the future work, we aim to explore techniques for quantum gate generation in quantum systems.

References

[1] Q Ansel et al. "Introduction to theoretical and experimental aspects of quantum optimal control". In: *Journal of Physics B: Atomic, Molecular and Optical Physics* 57.13 (June 2024), p. 133001. ISSN: 1361-6455.

[2] L V Lokutsievskiy, A N Pechen, and M I Zelikin. "Time-optimal state transfer for an open qubit". In: *Journal of Physics A: Mathematical and Theoretical* 57.27 (June 2024), p. 275302. ISSN: 1751-8121.